What, if anything, does quantum field theory explain?

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Fundamental particle physics aims to describe systems that exhibit the following two remarkable properties. First, they are invariant under the combined transformation of charge conjugation (C), spatial inversion (P), and time reversal (T): they are *CPT invariant*. Second, systems exhibiting Fermi-Dirac statistics have half-integer spin and systems exhibiting Bose-Einstein statistics have integer spin: the systems obey the *spin-statistics connection*. CPT invariance and the spin-statistics connection are empirically well-confirmed properties of systems of fundamental particles, and the received wisdom is that these features of the world can be explained by deriving them from the principles of relativistic quantum field theory. The derivations are referred to the CPT theorem and the spin-statistics theorem, respectively.

In *CPT Invariance and the Spin-Statistics Connection*, Jonathan Bain challenges this received wisdom and argues that in fact, we do not have an entirely adequate explanation of CPT invariance or the spin-statistics connection. His argument relies principally on the observation that there are different collections of principles that can be used to characterize what is meant by relativistic quantum field theory, each of which results in a different formalism. Moreover, each formalism affords a distinct derivation of the CPT theorem and the spin-statistics theorem. That there are multiple formalisms for relativistic quantum field theory is well-known, and typically foundational investigation proceeds by focusing on one particular formalism. The choice of formalism is then motivated by citing virtues of that particular approach. Bain, however, correctly notes that each formalism also has its own vices. Rather than focusing on a particular formalism, he systematically discusses the derivations in *each* of the formalisms, and the result is the most comprehensive foundational discussion of the CPT and spin-statistics theorems available in the literature.

Bain helpfully distinguishes between what he calls *purist* and *pragmatist* formalisms for relativistic quantum field theory. Purist formalisms are those that are captured axiomatically. The examples Bain considers are the Wightman axioms and the axioms of algebraic quantum field theory. Purist formalisms have the virtue of full mathematical rigor. The available models of the axioms are, however, physically unrealistic and thus face what Bain

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calls the problem of empirical import. The results captured in the purist formalisms do not apply to the empirically successful Standard Model of particle physics. Pragmatist formalisms avail themselves of the resources of perturbation theory to treat empirically interesting interactions. The examples Bain considers are the standard Lagrangian formalism and Weinberg's formalism. Through their use of perturbation theory these formalisms overcome the problem of empirical import and underwrite the empirical successes of the Standard Model. However, the use of perturbation theory also results in a number of mathematical problems in the foundations of the pragmatist formalisms. Particularly important for Bain's argument is that even after renormalization, the perturbative expansions diverge and thus do not define an exact model. He calls this the problem of existence.

The recognition that both pragmatist and purist formalisms face serious difficulties motivates Bain to consider whether any of them are actually capable of explaining CPT invariance and spin-statistics connection. As a first step toward answering this question, in Chapter One he provides an illuminating discussion of the logical relationships between the principles appealed to in the derivations of the CPT and spin-statistics theorems in each of the purist and pragmatist formalisms. The first important conclusion of this analysis is that the principles capturing the relativistic nature of the formalisms play distinct roles in their respective derivations of the CPT and spin-statistics theorems. In fact, Bain argues that Lorentz invariance is neither necessary nor sufficient for the derivation of the theorems.

In Chapter Two, Bain employs the argument of the first chapter to provide clarification to an important argument from the physics literature due to Greenberg.[1] One frequently finds references to Greenberg's argument which claim that it establishes that a violation of CPT invariance entails a violation of Lorentz invariance. In what is in my view the most important contribution of the book, Bain provides a carefully reconstruction of Greenberg's argument and considers a number of possible precisifications to it. The result of this analysis is that Greenberg's argument is only sound on the assumption that a solution to the existence problem is available. Stated alternatively, Bain shows that his argument is only valid and applicable to empirically adequate models at the level of perturbation theory. This is an important conclusion and I will return to its significance below.

The spin-statistics connection manifests itself in the non-relativistic regime as well as the domain of relativistic quantum field theory, and in Chapters Three and Four Bain considers how to go about explaining that fact. The method he employs is the same: he considers each of the available formalisms

[1] (Greenberg 2002)

and in this case explains why the spin-statistics connection and CPT invariance are not derivable properties of any of them. Moreover, he shows that this failure is not simply due to the failure of Lorentz invariance in the non-relativistic regime, and he isolates distinct reasons for the failure in each formalism. Lastly, for each formalism Bain identifies the relevant inter-theoretic relation connecting each non-relativistic formalism to its relativistic analog.

The analysis up to this point in the book shows why CPT invariance and the spin-statistics connection are derivable in each formalism for relativistic quantum field theory and why they fail to be derivable in each of the non-relativistic formalisms he discusses. This provides the required framework to address the question of whether or not we have an adequate explanation of CPT invariance and the spin-statistics connection in both the relativistic and non-relativistic domains. Bain considers the deductive-nomological, unificationist, causal, and, structuralist accounts of scientific explanation, and in each case he finds that the available mathematical results do not instantiate any of these patterns of explanation. Bain advocates that the presence of CPT invariance and the spin-statistics connection in the non-relativistic regime can be explained using the nature of the inter-theoretic relations with relativistic quantum field theory discussed in Chapters Three and Four, and an account of explanation due to Weatherall.[2] However, in the case of relativistic quantum field theory itself the appeal to inter-theoretic relations is of no help and Bain claims that in the relativistic regime we do not have an adequate explanation for the presence of CPT invariance and the spin-statistics connection.

Bain's reason for dismissing the explanations based on the pragmatist formalisms is that the relevant derivations are valid only at the level of perturbation theory and they suffer from the existence problem. Recall that this was also the problem with Greenberg's argument in Chapter Two. This raises an important question; namely, is there anything at all that we can explain with quantum field theory on Bain's view? The existence problem does not just raise itself in the context of the CPT and spin-statistics theorems. It is a problem for all of the results derived in pragmatist formalisms which convince us of the tremendous empirical success of the Standard Model of particle physics. In my view this motivates more careful attention to how it can be that results valid only at the level of perturbation theory can capture empirically adequate information about the world. I am optimistic that an account of this can be provided, but this falls beyond the task that Bain sets out for himself in the book.

[2] (Weatherall 2011)

Throughout the book, Bain's analysis is admirable for its clarity and the arguments provided for each of the conclusions are well-supported with details from the physics literature. There is much of value in this book, and it will quickly become an important reference for those with an interest in the foundations of quantum theories, and how physical theories discharge their explanatory duties.

References

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